Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

NICHOLAS J. HOFF

ON THE TRANSITION FROM AXISYMMETRIC TO MULTILOBED CREEP BUCKLING

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II

Department of Aeronautics and Astronautics
Stanford University
Stanford, California

ON THE TRANSITION FROM AXISYMMETRIC TO MULTILOBED CREEP BUCKLING

by

Nicholas J. Hoff

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ABSTRACT

Initial axisymmetric deviations from the exact circular cylindrical shape increase with time in consequence of creep when a uniformly distributed axial compressive load is acting on a thin-walled cylindrical shell. The increasing deformations are accompanied by the development of hoop stresses. When the hoop compression reaches a sufficiently high value, the shell can buckle elastically in agreement with Koiter's special theory.

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Theoretical Considerations

In a paper published in 1964 Samuelson [1] mentioned that circular cylindrical shells subjected to a uniformly distributed axial compressive load occasionally began to buckle in an axially symmetric manner, and later developed a multilobed buckling pattern. The tests were carried out with metal specimens under such conditions of stress and temperature that both elasticity and creep were responsible for significant parts of the deformations. During the tests the compressive load and the temperature were held constant.

In view of Koiter's papers [2,3] of 1945 and 1963 it appears worth while to follow up Samuelson's work both theoretically and experimentally. The theories of the Koiter papers were largely responsible for the explanation of the formerly perplexing behavior of perfectly elastic circular cylindrical shells which, as a rule, buckle under axial compressive stresses much lower than the critical stress of the classical small-displacement theory. Koiter showed that this difference is a consequence of the great sensitivity of the system to initial deviations from the exact circular cylindrical shape. In his so-called special theory [4] he assumed the existence of small initial axisymmetric deviations from the exact shape which vary sinusoidally with the axial coordinate. When the compressive load is applied, the amplitude of the deviations increases and the increasing displacements are accompanied by the appearance of hoop stresses in the shell. The destabilizing effect of the hoop compression is an important element of the problem; it can lead to the development of a multilobed pattern of buckling before the amplitude of the axisymmetric deformations increases beyond all bounds according to the small-displacement theory.

But in the presence of creep deformations the conditions are quite similar to those prevailing when the shell is perfectly elastic. The only change is that the initial deviation amplitude

of the shell keeps increasing with time in consequence of creep under a constant applied load while in the case of the perfectly elastic shell the initial amplitude is constant and the applied load keeps increasing during the loading process. Thus the role played by the critical load of elastic stability theory is taken over by the critical time in the presence of creep: it is the time when, under a prescribed constant load, the initial deviation amplitude of the axisymmetric deviations reaches the value under which the shell becomes elastically unstable in the presence of multilobed disturbances.

The growth in consequence of creep of initial axisymmetric deviations from the exact circular cylindrical shape was calculated by the author in 1968 [5]. His implicit expression is reproduced here:

$$t = 0.294\, t_E \,\ell n \frac{x_{fin}^2(1.18+x_o^2)}{x_o^2(1.18+x_{fin}^2)} \tag{1}$$

Here

$$x = w/d \tag{2}$$

and w is the amplitude of the axisymmetric deviations from the exact circular cylindrical shape at any time t while d is the distance between the faces of the sandwich model of the real solid wall of the shell. The equivalence was established on the basis of Rabotnov's [6] suggestion:

$$d = [n/(2n+1)]^{n/(n+1)} h \tag{3}$$

In this equation h is the thickness of the real solid wall of the cylindrical shell and n is the exponent in the uniaxial form of the creep law governing the creep deformations of the material of the shell:

$$\dot{\epsilon} = k\sigma^n \tag{4}$$

where ϵ is the uniaxial tensile strain, σ the corresponding stress, k an empirical constant, and the dot over the ϵ indicates differentiation with respect to time. Since in the derivation of (1) n was assumed to be 3, the value of d as given in (3) becomes

$$d = 0.528h \tag{5}$$

The subscripts 0 and fin refer to the initial and final values of the deviations characterizing the state of the shell at the moments of load application and multilobed elastic buckling, respectively. Finally, t_E is the Euler time defined as

$$t_E = \epsilon_E/\dot{\epsilon}_{nom} \tag{6}$$

where ϵ_E is the strain at which a perfectly elastic axially compressed circular cylindrical shell buckles according to the classical small-displacement theory:

$$\epsilon_E = 0.6(h/a) \tag{7}$$

a is the mean radius of the shell, and $\dot{\epsilon}_{nom}$ is the nominal creep strain rate of the perfect shell

$$\dot{\epsilon}_{nom} = k(P/2\pi ah)^n \tag{8}$$

with P the value of the compressive load applied to the shell (positive when compressive).

Equation (1) was derived on the basis of the assumption that the shape of the initial deviations is sinusoidal with a wavelength equal to that wavelength at which the creep deformations increase most rapidly. For all practical purposes this is also equal to the critical wavelength of axisymmetric elastic buckling. It has already been mentioned that in the analysis n was taken as 3; this restriction was discarded in a paper by Honikman and the author [7] in which results were obtained for $n = 5,7,$ and 9 , and the results

were extended in an approximate manner to $n = 29$. The expression presented in the paper is the same as (1) except that the values of the numerical constants vary with n .

In the analysis of [5] it was assumed that the material of the shell is capable only of steady creep deformations. When it also deforms elastically in accordance with Hooke's linear law, (1) must be replaced with (see [8])

$$t = 0.294 \frac{\sigma_E - \sigma}{\sigma_E} t_E \ln \frac{x_{fin}^2 (1.18 + x_o^2)}{x_o^2 (1.18 + x_{fin}^2)} \tag{9}$$

where σ_E is the critical stress of the classical linear theory

$$\sigma_E = 0.6E(h/a) \tag{10}$$

and σ is the intensity of the applied compressive stress

$$\sigma = P/2\pi ah \tag{11}$$

(positive when compressive).

In the presence of both elastic and steady creep deformations x_o is the amplitude of the initial deviations after load application

$$x_o = \frac{\sigma_E}{\sigma_E - \sigma} x_{oo} \tag{12}$$

where

$$x_{oo} = w_{oo}/d \tag{13}$$

is the nondimensional deviation amplitude before load application.

The critical value w_{cr} of the initial deviations is given by Koiter's formula

$$x_{cr} = 0.76 \frac{(1-\rho)^2}{\rho} \tag{14}$$

obtained from the original expression in [4], namely

$$\psi = [4/27(1-\nu^2)]^{1/2}[(1-\rho)^2/\rho] \tag{15}$$

where

$$\psi = w/h \tag{16}$$

and ρ is the ratio of the stress at which multilobed buckling occurs to the critical stress σ_E of the classical theory. The numerical factors were obtained on the basis of the assumption that $\nu = 0.3$.

When the intensity σ of the applied compressive stress and the geometric and mechanical properties of the shell are specified, the ratio of σ to σ_E can again be denoted ρ. The critical value of x for multilobed buckling corresponding to this value of ρ can be computed from (14). This value represents an initial deviation before load application; hence it must be multiplied by $\sigma_E/(\sigma_E-\sigma)$ before it is substituted for x_{fin} in (9). Thus one obtains the critical time t_{mult} for multilobed buckling under the simultaneous action of the axial and the hoop stresses as

$$t_{mult} = 0.294\,\frac{\sigma_E-\sigma}{\sigma_E}\,t_E\,\ln\frac{[\sigma_E/(\sigma_E-\sigma)]^2 x_{cr}^2(1.18+x_o^2)}{x_o^2(1.18+[\sigma_E/(\sigma_E-\sigma)]^2 x_{cr}^2)} \tag{17}$$

Axisymmetric creep buckling occurs under the same stress σ at the critical time t_{axi} which is obtained from (9) when x_{fin} is made to increase beyond all bounds:

$$t_{axi} = 0.294\,\frac{\sigma_E-\sigma}{\sigma_E}\,t_E\,\ln\frac{1.18+x_o^2}{x_o^2} \tag{18}$$

The ratio R of the two critical times is therefore

$$R = t_{mult}/t_{axi} \tag{19}$$

This expression simplifies substantially when both x_o and x_{cr} are small:

$$R = \frac{\ell n(x_{cr}/x_{oo})}{\ell n(1.086/x_o)} = \frac{\log(x_{cr}/x_{oo})}{\log(1.086/x_o)} \qquad (20)$$

when $[\frac{\sigma_E}{\sigma_E-\sigma}]^2 x_{cr}^2 << 1.18$ and $x_o^2 = [\frac{\sigma_E}{\sigma_E-\sigma}]^2 x_{oo}^2 << 1.18$

A numerical example will show the significance of the reduction. Let us take

$$\rho = 0.8 \qquad x_{oo} = 10^{-3}$$

From (12) we obtain

$$x_o = 0.005$$

and from (14) we have

$$x_{cr} = 0.038$$

It follows from (20) that

$$R = \log 38/\log 217.2 = 1.579/2.337 = 0.676$$

A large reduction in the critical time can be expected to occur in consequence of multilobed elastic buckling whenever ρ is high. When ρ is very small, x_{cr}^2 can be much larger than 1.18 with the result that R is approximately equal to unity. In such cases it is not too likely that multilobed buckling can be observed in the experiment. On the other hand it is to be remembered that Koiter's equation was derived on the assumption that $\psi < 1$; for this reason any conclusion based on x_{cr} values much greater than one is highly speculative.

Experiments

A doctoral student of the author, Michel Benoit, recently carried out creep buckling experiments with 31 nickel specimens [9]. The specimens were manufactured very carefully by R. L. Sendelbeck who made use of the electroplating technique. The specimens had a diameter of 1.225 in., a length of 4.40 in. and wall thicknesses ranging from 12.7×10^{-3} in. to 40×10^{-3} in. The tests were carried out under a constant load applied through a lever; the specimens were placed inside an oven whose temperature was maintained at 650°F.

A special feature of the tests was an interruption of the loading once or twice for the purpose of photographing the specimen and measuring accurately its deformations. In this manner it was possible to determine that the buckling of many of the specimens began axisymmetrically and continued and terminated in a multilobed fashion.

Figures 1 and 2 are photographs of typical axisymmetric and multilobed buckling patterns.

Concluding Remarks

It has been shown that the effect of the hoop compression developing in consequence of axially symmetric creep deformations can lead to a reduction in the critical time of axially compressed thin-walled circular cylindrical shells. This behavior is analogous to the reduction in the critical axial stress of perfectly elastic shells in consequence of the same hoop stresses as described by Koiter in 1945. In both cases the initial deformations are axially symmetric while the final buckling pattern is multilobed.

The two figures of the paper show these two patterns. They are photographs of specimens of the present series; they were easy to take because the creep deformations develop slowly. They may be useful as a confirmation of the correctness of the physical

basis of the Koiter theory valid for perfectly elastic shells; in the case of these shells photography of the changes in the buckling pattern is difficult as the transition from the axisymmetric state to the multilobed state takes place very rapidly.

One might argue that the change to the multilobed pattern should be a snap-through phenomenon even in the presence of creep because it is caused by the appearance of sufficiently high elastic hoop stresses. However, the end shortening versus time curves show no discontinuity of the tangent even though they become quite steep, as a rule, when the shell develops large deformations. It is likely that the absence of the discontinuity is a consequence of the presence of small initial multilobed components of the deviations from the exact circular cylindrical shape.

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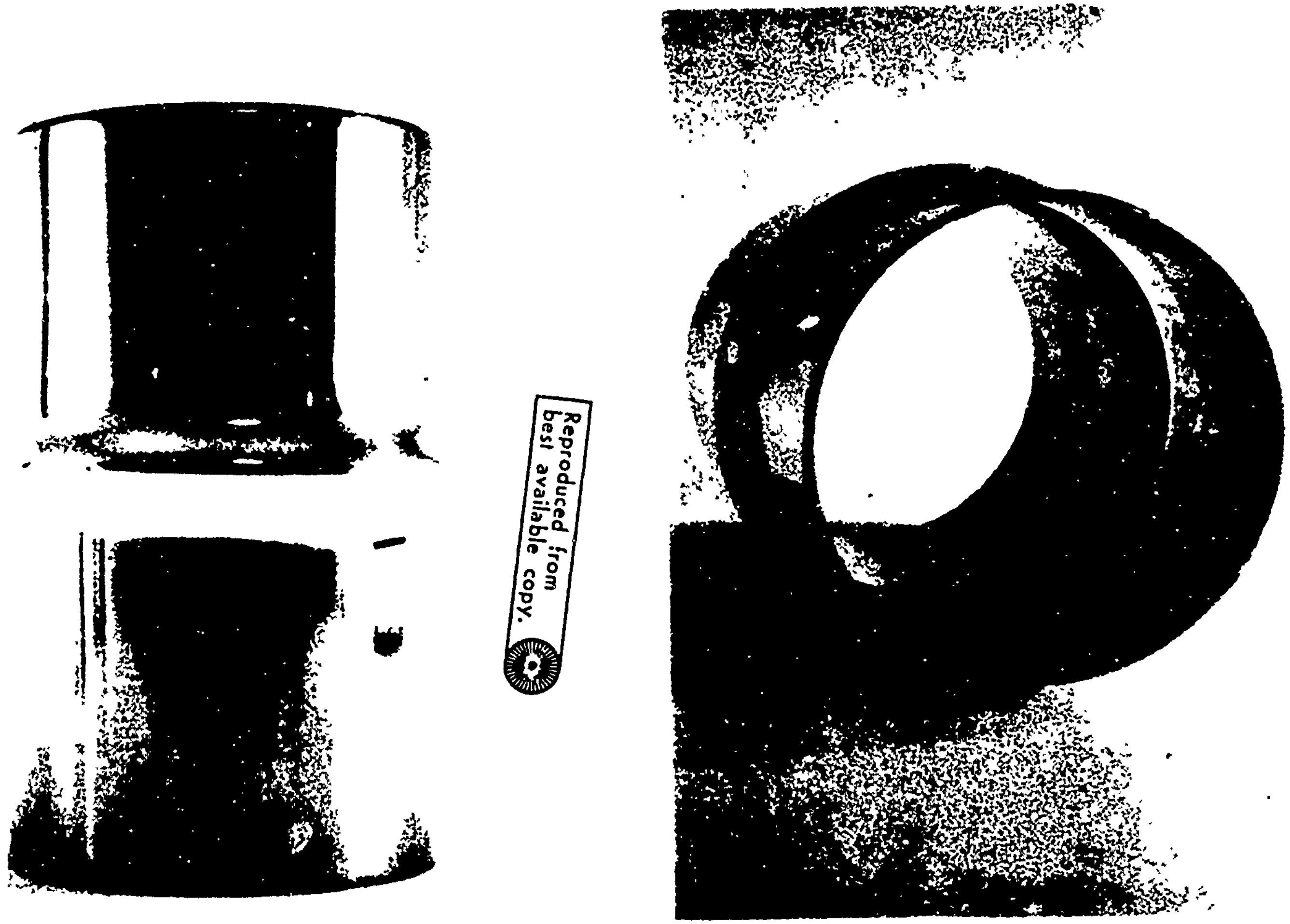

FIG. 1. AXISYMMETRIC CREEP BUCKLING.

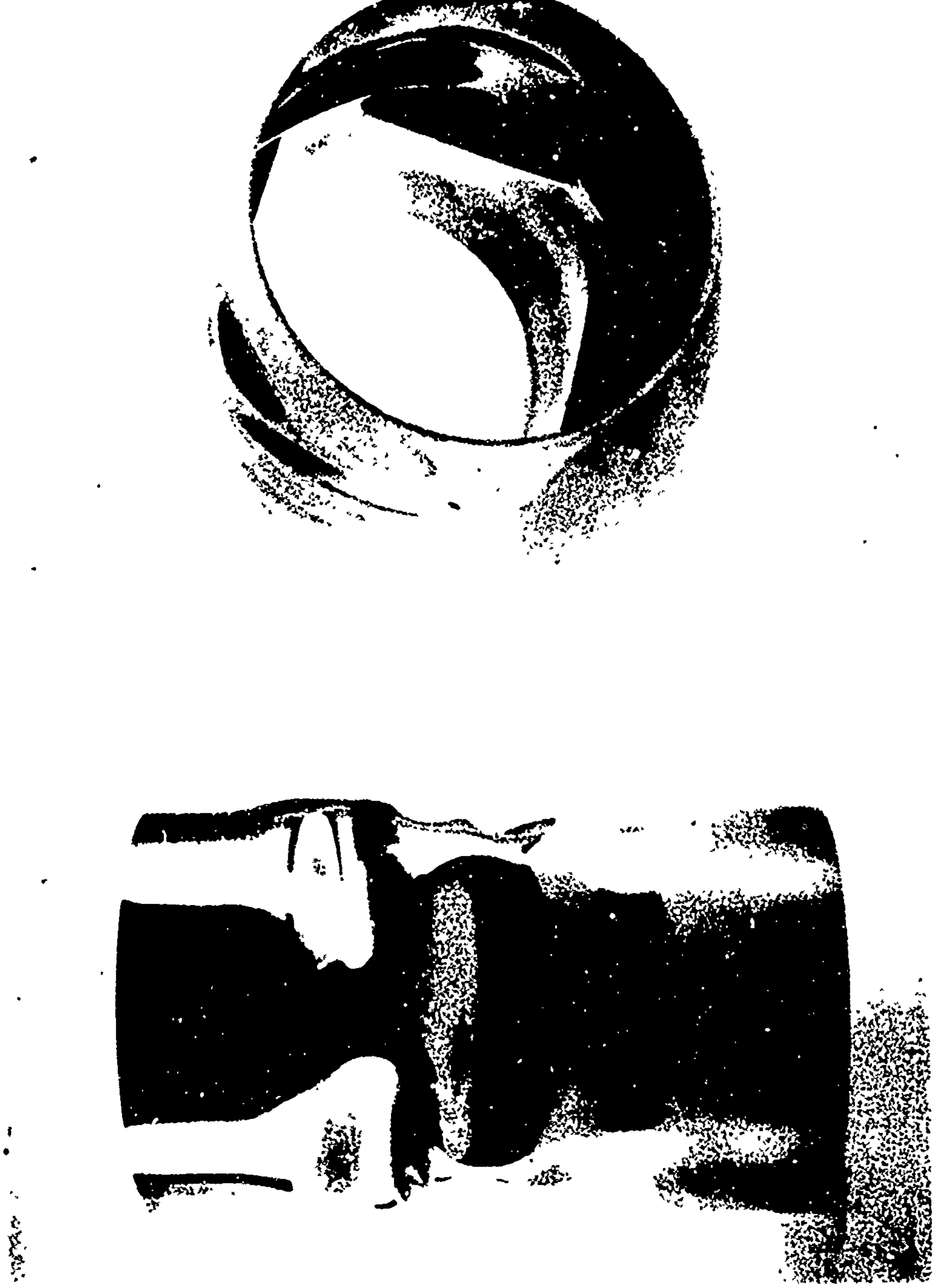

FIG. 2. MULTILOBED CREEP BUCKLING.